Pre Board Examination 2020-21 (SET – B)

Class: XII

Subject: Mathematics

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains two **parts A and B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks
2. **Part-A** has Objective Type Questions and **Part -B** has Descriptive Type Questions
3. Both Part A and Part B have choices.

Part – A:

1. It consists of two sections**- I and II**.
2. Section **I** comprises of 16 very short answer type questions.
3. Section **II** contains **2** case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.

Part – B:

1. It consists of three sections**- III, IV and V.**

2. Section **III** comprises of 10 questions of **2 marks** each**.**

3. Section **IV** comprises of 7 questions of **3 marks** each**.**

4. Section **V** comprises of 3 questions of **5 marks** each**.**
Internal choice is provided in **3** questions of Section –III, **2** questions of Section- IV and **3** questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part – A

Section I

All questions are compulsory. In case of internal choices attempt any one.

1. Let A = {1,2,3}, B = { 4,5,6,7 } and let f = { (1,4). (2,5) , (3,6) } be a function from A to B. State whether f is onto or not.

OR

State the reason for the relation R in the set { 1,2,3} given by R = { (1,3) , (3,1)} not to be transitive.

2. Show that the function f : R→R, given by f(x) = cosx , $\forall x \in R$, is neither one-one nor onto.
3. Find the smallest equivalence relation R on set A = {1,2,3, 4}.

OR

For the set A = {1,2,3} define a relation R in the set A as follows : R = { (1,1), (2,2) , (3,3) , (1,3) }. Find the ordered pair to be added to make it the smallest equivalence relation.

4. If A is a matrix of order 2*3 and B is a matrix of order 3*5 , what is the order of (AB)' .
5. If A is any square matrix of order 3*3 such that I A I = -5, then find the value of I A. Adj A I.

OR

If A and B are square matrices of order 3 each, I A I = 3 and I B I = 2, then find the value of I 3AB I.

6. If A = [a_{ij}] is a square matrix of order 2*2, such that I A I = -15 and C_{ij} represents the cofactor of a_{ij} , then find the value of a_{21} C_{21} + a_{22} C_{22}.

7. $\int \frac{e^x\ (1+x)}{cos^2\ (\ e^x\ x)}$ dx

OR

$$\int_0^{\pi} sin^3\ x\ dx$$

8. Find the area of the region bounded by the curve x = 2y + 3 and the lines y = 1, y = -1 and y - axis.

9. Find the general solution of e^x cosy dx - $e^x\ siny\ dy$ = 0

OR

If cosx is an integrating factor of the differential equation $\frac{dy}{dx} + Px = Q$, find P.

10. If 2i – 3j + 4k and ai + 6j – 8k are collinear, then find the value of a.

11. Find the magnitude of projection of (2i – j + k) on (1 -2j + 2k).

12. Find the value of the expression I a X b I^2 + (a . b)^2.

13. P is a point on the line segment joining the points (3,2,-1) and (6,2,-2). If x coordinate of P is 5, then find its z coordinate.

14. Find the distance between the parallel planes r. (2i –j -2k) = 6 and r. (6i -3j – 6k) = 27.

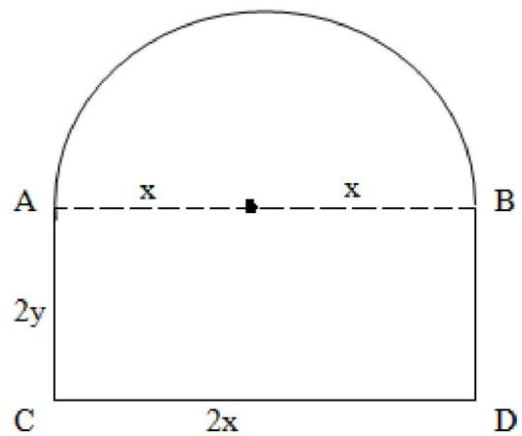
15. If A and B are two independent events such that P(A) = $\frac{3}{5}$ and P(B) = $\frac{4}{9}$, then find P(A' ∩ B') .

16. A couple has 2 children. Find the probability that both are boys, if it is known that elder child is a girl.

Section II

Both the Case study based questions are compulsory. Attempt any 4 sub parts from each question (17-21) and (22-26). Each question carries 1 mark

17. Mr Shashi, who is an architect, designs a building for a small company. The design of window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangle which is surmounted by a semi-circular opening. This window is having a perimeter of 10 m as shown below :



Based on the above information answer the following :

(i) If 2x and 2y represents the length and breadth of the rectangular portion of the windows, then the relation between the variables is

(a) $4y - 2x = 10 - \pi$

(b) $4y = 10 - (2 - \pi)x$

(c) $4y = 10 - (2 + \pi)x$

(d) $4y - 2x = 10 + \pi$

(ii) The combined area (A) of the rectangular region and semi-circular region of thewindow expressed as a function of x is

(a) $A = 10x + (2 + \frac{\pi}{2})\, x^2$

(b) $A = 10x - (2 + \frac{\pi}{2})\, x^2$

(c) $A = 10x - (2 - \frac{\pi}{2})\, x^2$

(d) $A = 4xy + \frac{1}{2}\pi x^2$, where $y = \frac{5}{2} + \frac{1}{4}(2 + \pi)x$

(iii) The maximum value of area of the whole window, A is

(a) $A = \frac{50}{4+\pi}$ cm^2

(b) $A = \frac{50}{4+\pi}$ m^2

(c) $A = \frac{100}{4+\pi}$ m^2

(d) $A = \frac{50}{4-\pi}$ m^2

(iv) The owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible. For this to happen, the length of rectangular portion of the window should be

(a) $\frac{20}{4+\pi}$ m

(b) $\frac{10}{4-\pi}$ m

(c) $\frac{4}{10+\pi}$ m

(d) $\frac{100}{4+\pi}$ m

(v) In order to get the maximum light input through the whole window, the area (in sq. m) of the only semi-circular opening of the window is

(a) $\frac{100\pi}{(4+\pi)^2}$

(b) $\frac{50\pi}{4+\pi}$

(c) $\frac{50\pi}{(4-\pi)^2}$

(d) Same as the area of rectangular portion of the window.

18. There are three categories of students in a class of 60 students :

A : Very hard working students
B : Regular but not so hard working
C : Careless and irregular.

It's known that 10 students are in category A, 30 in category B and rest in category C. It is also found that probability of students of category A, unable to get good marks in the final year examination is, 0.002, of category B it is 0.02 and of category C, this probability is 0.20.

Based on the above information answer the following :
(i) If a student selected at random was found to be the one who could not get good marks in the examination, then the probability that this student is of category C is

(a) $\frac{201}{231}$

(b) $\frac{200}{231}$

(c) $\frac{21}{231}$

(d) $\frac{31}{231}$

(ii) Assume that a student selected at random was found to be the one who could not get good marks in the examination. Then the probability that this student is either of category A or of category B is

(a) $\frac{31}{231}$

(b) $\frac{200}{231}$

(c) $\frac{201}{231}$

(d) $\frac{21}{231}$

(iii) The probability that the student is unable to get good marks in the examination is

(a) $\frac{231}{300}$

(b) $\frac{231}{3000}$

(c) $\frac{770}{1000}$

(d) 0.007

(iv) A student selected at random was found to be the one who could not get good marks in the examination. The probability that this student is of category A is

(a) $\frac{1}{231}$

(b) $\frac{200}{231}$

(c) $\frac{230}{231}$

(d) None of these

(v) A student selected at random was found to be the one who could not get good marks in the examination. The probability that this student is **NOT** of category A is

(a) 0

(b) $\frac{230}{231}$

(c) $\frac{21}{231}$

(d) 1

PART 'B'

SECTION III

19. Find the value of $\tan^{-1}(\frac{-1}{\sqrt{3}})$ + $\cot^{-1}(\frac{-1}{\sqrt{3}})$ + $\tan^{-1}(\sin(\frac{\pi}{2}))$.

20. If A = $\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and kA = $\begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then find the values of k, a and b.

OR

If A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, f(x) = $x^2 - 2x - 3$, show that f(A) = O.

21. For what value of k is the function f(x) = $\begin{cases} x\sin\frac{1}{x} & , \quad if\ x \neq 0 \\ k, & if\ x = 0 \end{cases}$ continuous at x = 0.

22. Find the local maximum and minimum points of f(x) = sinx + cosx, x ∈ $0 < x < 2\pi$

23. Find $\int \frac{sin^3 x + cos^3 x}{sin^2 x\, cos^2 x}$ dx

OR

Evaluate : $\int_{-2}^{2}(x^3 + sin^7 x)\, dx$

24. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

25. Can $y = ax + \frac{b}{a}$ be a solution of the following differential equation?

$Y = x\frac{dy}{dx} + \frac{b}{dy/dx}$

26. Find a unit vector perpendicular to both the vector a and b , where a = I – 7j + 7k and b = 3i -2j + 2k.

27. Write the vector equation of the plane passing through the point (a,b,c) and parallel to the plane r. (I + j+ k) = 2.

28. If A and B are two independent events, prove that A' and B' are also independent.

OR

If P(A)= 0., P(B) = p, P(A∪ B) = 0.6 and A and B are given to be independent events, find the value of p.

Section IV

All questions are compulsory. In case of internal choices attempt any one.

29. Let A = {x ∈ Z : 0 ≤ x ≤ 15}. Show that R = { (a,b) : a,b ∈ A, I a-b I is a multiple of 3} is an equivalence relation. Also write the equivalence class [2].

30. If $x^m y^n = (x + y)^{m+n}$, prove that $\frac{d^2y}{dx^2} = 0$.

31. If $\sqrt{(1 - x^2)} + \sqrt{(1 - y^2)} = a(x - y)$, $|x| < 1$, $|y|<1$, show that $\frac{dy}{dx} = \sqrt{\frac{(1-y^2)}{(1-x^2)}}$.

OR

If x = sec$\theta - cos\theta$, y = sec$^n\theta - cos^n\theta$, then prove that $(x^2 + 4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2 + 4)$.

32. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

33. Find : $\int \frac{x}{(2+x^2)(4+x^4)} dx$

34. Using integration, find the area of the region { (x,y) : $x^2 + y^2 \le 1, x + y \ge 1, x \ge 0, y \ge 0$}.

OR

Using integration find the area of the region bounded by the curve y = I x + 3 I above x- axis and between x = -6 to x = 0.

35. Check whether the given differential equation is homogeneous or not:

$x^2 \frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right), x \neq 0$. Find the general solution of the given differential equation using substitution y = vx.

Section V

All questions are compulsory. In case of internal choices attempt any one.

36. if A = $\begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$, find A^{-1}. Hence solve the system of equations

3x + 3 y + 2z = 1

x + 2y = 4

2x - 3Y - z = 5

OR

Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

x – y + 2z = 1

2y -3z =1

3x -2y + 4z = 2

37. Find the shortest distance between the lines :

r = (I + 2 j + 3k) + μ ($2i + 3j + 4k$) and r = (2i + 4 j + 5k) + ρ $(4i + 6j + 8k)$.

OR

Find the vector equation of the plane determined by the points A(3 , -1, 2), B(5, -2, 4) and C(-1, -1, 6). Hence , find the distance of the plane, thus obtained from the origin.

38. Solve the following LPP graphically

Maximize Z = 50x + 60y

Subject to the following constraints

2X + y ≤ 18

x +2 y ≤ 12

x + 3y ≤ 15

X,y ≥ 0

OR

Solve the following LPP graphically

Minimize Z = 100x + 50y

Subject to constraints

X + y ≤ 300

3X + y ≤ 600

Y – x ≤ 200

X,y ≥ 0