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AN EDUCATIONAL PORTAL FOR MATH SCHOLARS

PLEASURE TEST SERIES XII – 02

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Time Allowed : 3 Hours **Max. Marks : 80**

General Instructions :

1. This question paper contains two **Parts A and B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks.
2. **Part A** has Objective Type Questions and **Part B** has Descriptive Type Questions.
3. Both Part A and Part B have choices.

Part A :

1. It consists of two sections - **I and II**.
2. Section I comprises of 16 very short answer type questions.
3. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.
4. Internal choice is provided in **5** questions of Section - I. Moreover internal choices have been given in both questions of Section - II as well.

Part B :

1. It consists of three sections - **III, IV and V**.
2. Section III comprises of 10 questions of **2 marks** each.
3. Section IV comprises of 7 questions of **3 marks** each.
4. Section V comprises of 3 questions of **5 marks** each.
5. Internal choice is provided in **3** questions of Section - III, **2** questions of Section - IV and **3** questions of Section - V. You have to attempt only one of the alternatives in all such questions.

PART - A

Section I

Questions in this section carry 1 mark each.

Q01. Evaluate $\int x \sin x dx$.

OR

Evaluate $\int \sin^3 x \cos x dx$.

Q02. Check if the function $f : N \to N$ defined by $f(x) = 2 - 3x$ is one-one or not?

Q03. Write the general solution of following differential equation :

$$\frac{dx}{dy} = 3^{y-x}.$$

Q04. Find the direction cosines of a line which makes equal angles with the coordinate axes.

OR

Write the Cartesian equation of the line $\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + \hat{j} - 2\hat{k})$.

Q05. If the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1,2), (2,1), (1,1)\}$. Then state if R is transitive relation, justify your answer.

Q06. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then find the probability of getting exactly one red ball.

OR

If $P(X) = \frac{2}{5}$, $P(Y) = \frac{3}{10}$, $P(X \cap Y) = \frac{1}{5}$, then find the value of $P(X' | Y')$.

Q07. What is the acute angle between $\hat{i} - \hat{j} + \hat{k}$ and $-2\hat{i} + \hat{j} - \hat{k}$?

Q08. Simplify : $\sec\theta \begin{pmatrix} \tan\theta & \sec\theta \\ \sec\theta & -\tan\theta \end{pmatrix} - \tan\theta \begin{pmatrix} \sec\theta & \tan\theta \\ \tan\theta & -\sec\theta \end{pmatrix}$.

Q09. If A is an order 3 matrix, and $A'A = I$ then, find the value of $\left|2(\text{adj}.A)^{-1}\right|$.

OR

Find $\begin{vmatrix} \log_3 512 & \log_2 3 \\ \log_3 8 & \log_2 9 \end{vmatrix}$.

Q10. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j}$ denote the sides of a triangle, find the area of this triangle.

Q11. If E and F are events such that $0 < P(F) < 1$, then write the value of $P(E | F) + P(E' | F)$.

Q12. Find the vector equation of a line passing through the point $(-1, 5, 4)$ and perpendicular to the plane $z = 0$.

OR

The z-coordinate of a point on the line joining A(5, 1, –2) and B(2, 2, 1) is '–1'. Find its x-coordinate.

Q13. If a matrix has 18 elements, then how many possible orders it can have?

Q14. Let a relation R in a set A contains $(a_1, a_2) \in R$. If R is a symmetric relation, then write the element which must be in R.

Q15. If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then find the value of λ.

Q16. Find the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{x^2} \tan\left(\frac{1}{x}\right) dx$, where $x \neq 0$.

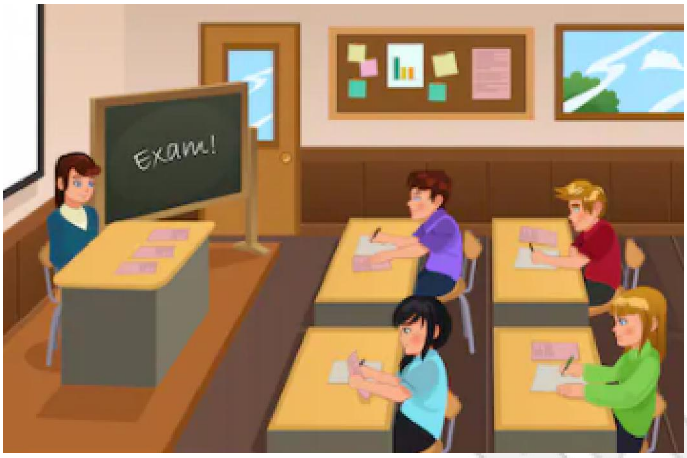
Section II

Questions in this section carry 1 mark each.

Both the Case study based questions are compulsory. **Attempt any 4 sub-parts from each question 17 (i-v) and 18 (i-v).**

Q17. In a city school, all class XII students have Mathematics and Biology as their main subjects, apart from three other subjects which include one language.

The school conducted pre-board examination for class XII. In the examination, it is observed that 30% of the students failed in Biology, 25% failed in Mathematics and 12% failed in both Biology and Mathematics. A student is selected at random from the school.



Using the information given above, answer the following :

(i) The probability that the selected student has failed in Biology, if it is known that he has failed in Mathematics, is
(a) 30%
(b) 25%
(c) 48%
(d) 12%

(ii) The probability that the selected student has failed in Mathematics, if it is known that he has failed in Biology, is
(a) 48%
(b) 40%
(c) 88%
(d) 12%

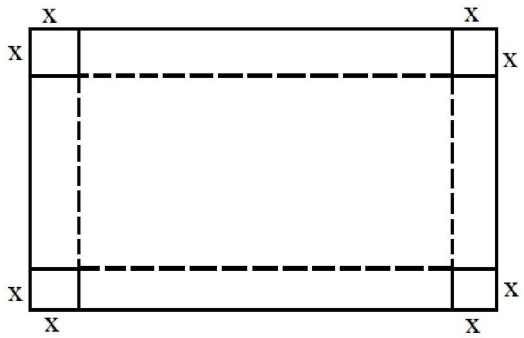
(iii) The probability that the selected student has failed in at least one of the two subjects, is
(a) 88%
(b) 40%
(c) 43%
(d) 60%

(iv) The probability that the selected student has passed in at least one of the two subjects, is
(a) 25%
(b) 88%
(c) 22%
(d) 43%

(v) The probability that the selected student has passed in Mathematics, if it is known that he has failed in Biology, is
(a) $\frac{1}{5}$
(b) $\frac{2}{5}$
(c) $\frac{3}{5}$

(d) $\frac{4}{5}$

Q18. An online retail company ships its products in the cartons.
Each of the cartons is made by a rectangular sheet of fiberboard with dimensions of 8 m by 3 m. While making the carton, equal squares of side length 'x' metres are cut-off from each corner of the rectangular sheet of fiberboard. After that, the resulting flaps are folded up to form the carton.
See the figure given below to identify the sides of the carton.



Based on the above information, answer the following :

(i) The volume V of the carton is given by $V = f(x)$, then $f(x)$ equals

(a) $22x - 24x^2 + 4x^3$

(b) $\frac{1}{24} - \frac{1}{22}x + \frac{1}{4}x^3$

(c) $24x + 22x^2 - 4x^3$

(d) $24x - 22x^2 + 4x^3$

(ii) Consider the function $f(x)$ obtained in **(i)**. Then $f'(x)$ equals

(a) $24 + 44x - 12x^2$

(b) $24 - 44x + 12x^2$

(c) $8x - 22$

(d) $22 - 48x + 12x^2$

(iii) The value of x (in metres) for which the volume V of the carton is maximum, is

(a) 3

(b) $\frac{2}{3}$

(c) both (a) and (b)

(d) none of (a) and (b)

(iv) What is the length (in metres) of the carton formed, for maximum value of V?

(a) $\frac{1}{3}$

(b) $\frac{5}{3}$

(c) $\frac{20}{3}$

(d) $\frac{2}{3}$

(v) What is the maximum volume of the carton formed?

(a) $\frac{200}{7}$ m^3

(b) 81 m^3

(c) $\frac{200}{27}$ cm^3

(d) $\frac{200}{27}$ m^3

PART - B

Section III

Questions in this section carry 2 marks each.

Q19. If $x = at^2, y = 2at,$ then find $\frac{d^2y}{dx^2}$ at $t = \frac{1}{2}$.

Q20. Find $\int \frac{1}{\sqrt[2]{x} + \sqrt[3]{x}} dx$.

OR

Evaluate $\int \frac{dx}{1 + 8\sin^2 x + \cos^2 x}$.

Q21. A wallet contains 3 silver and 6 copper coins and a second wallet contains 4 silver and 3 copper coins. If a coin is drawn at random from one of the two wallets, find the probability that it is a silver coin.

Q22. Simplify : $\sin^{-1}(2x\sqrt{1-x^2}),\ \frac{1}{\sqrt{2}} \le x \le 1$.

Q23. Let A be a square matrix of order 'k' such that $\begin{bmatrix} 3 & -2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ k \\ -5 \end{bmatrix} = O$, where O is the null matrix and det.(A) is 7. Then, write the value of det.(2 adj.A).

OR

If $A = \begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find scalar k so that $A^2 + I = kA$.

Q24. Solve : $\left(\frac{e^{-2\sqrt{x}} - y}{\sqrt{x}} \right) \frac{dx}{dy} = 1,\ x \ne 0$.

Q25. Find the distance between the planes $\vec{r}.(2\hat{i} - 4\hat{j} + 2\hat{k}) = 3$ and $\vec{r}.(\hat{i} - 2\hat{j} + \hat{k}) + 5 = 0$.

Q26. Find the points on the curve $y = x^3 - 3x^2 - 4x$ at which the tangent lines are parallel to the line $8x + 2y - 3 = 0$.

Q27. Find the area bounded by $y^2 = x,\ y = 5$, and y-axis.

Q28. Find all the unit vectors perpendicular to the plane PQR, where P(3, –1, 2), Q(1, –1, –3) and R(4, –3, 1).

OR

Given that $|\vec{a}|=1$ and $|\vec{b}|=1$. Find the angle between the vectors $\vec{a}$ and $\vec{b}$, so that $\sqrt{3}\,\vec{a}+\vec{b}$ is a unit vector.

Section IV

Questions in this section carry 3 marks each.

Q29. Find the intervals in which the function $f(x)=(x-1)^3(x-2)^2$ is
(a) strictly increasing
(b) strictly decreasing.

Q30. If $y=e^{x^2\cos x}+(\cos x)^x$, then find $\frac{dy}{dx}$.

OR

Find $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{3}$, if $y=3\cos t-\cos 3t$ and $x=3\sin t-\sin 3t$.

Q31. Find the area bounded by the lines $y=\big||x|-1\big|$ and the x-axis. Use integration.

Q32. Let N be the set of natural numbers and R be the relation on $N\times N$ defined by $(a, b)\ R\ (c, d)$ iff $ad=bc$ for all $a, b, c, d\in N$. Show that R is an equivalence relation.

Q33. Evaluate $\int \frac{x^3+1}{x^3-x}\,dx$.

OR

Evaluate $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x\,dx}{1+\sin x}$.

Q34. Solve the differential equation $(x\sqrt{x^2+y^2}-y^2)\,dx+xy\,dy=0$.

Q35. If $f(x)=\begin{cases} x+a\sqrt{2}\sin x, & 0\le x<\frac{\pi}{4} \\ 2x\cot x+b, & \frac{\pi}{4}\le x\le\frac{\pi}{2} \\ a\cos 2x-b\sin x, & \frac{\pi}{2}<x\le\pi \end{cases}$ is continuous in $x\in[0,\pi]$, then find the value of a and b.

Section V

Questions in this section carry 5 marks each.

Q36. For the matrix $A=\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, find the value of det. A and adj.A.

Hence, show that $A(\text{adj}.A)=|A|I_3=(\text{adj}.A)A$.

OR

If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1}.

Using the inverse of matrix A, solve the following system of equations :

$x + 2y - 3z = 6,\ 3x + 2y - 2z = 3,\ 2x - y + z = 2$.

Q37. Solve the following linear programming graphically :
Minimize $Z = x - y$
Subject to constraints
$x + y \le 9$,
$x + y \ge 5$,
$x \le 7$,
$y \le 6$,
$x \ge 0$,
$y \ge 0$.
At what point, the minimum value of Z occurs?

OR

Using graphical method, solve the following linear programming :
Maximize : $Z = 150x + 250y$
Subject to :
$x + y \le 35$,
$35x + 70y \le 1750$,
$x \ge 0$,
$y \ge 0$.
Also, write the x and y coordinate of the point at which Z_{max} occurs.

Q38. Find the vector and Cartesian equations of the line which is perpendicular to the lines with equations $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and, $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1, 1, 1). Hence, write the direction cosines of the line thus obtained.

OR

Find the distance of the point $P(-2, -4, 7)$ from the point of intersection Q of the line $\vec{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 6$. Also write the vector and Cartesian equation of the line PQ.