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AN EDUCATIONAL PORTAL FOR MATH SCHOLARS

PLEASURE TEST SERIES XII – 01

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Time Allowed : 3 Hours **Max. Marks : 80**

General Instructions :

1. This question paper contains two **Parts A and B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks.
2. **Part A** has Objective Type Questions and **Part B** has Descriptive Type Questions.
3. Both Part A and Part B have choices.

Part A :

1. It consists of two sections - **I and II**.
2. Section I comprises of 16 very short answer type questions.
3. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.
4. Internal choice is provided in **5** questions of Section - I. Moreover internal choices have been given in both questions of Section - II as well.

Part B :

1. It consists of three sections - **III, IV and V**.
2. Section III comprises of 10 questions of **2 marks** each.
3. Section IV comprises of 7 questions of **3 marks** each.
4. Section V comprises of 3 questions of **5 marks** each.
5. Internal choice is provided in **3** questions of Section - III, **2** questions of Section - IV and **3** questions of Section - V. You have to attempt only one of the alternatives in all such questions.

PART - A

Section I

Questions in this section carry 1 mark each.

Q01. Find the number of non-reflexive relations defined on a set with three elements.

Q02. Let $f : R \to (-\pi, 0)$ be defined as $f(x) = \cot^{-1} x$.
Then find the value of $f(-1)$.

OR

If $\sin^{-1} x = \frac{\pi}{5}, x \in (-1,1)$ then, find $\cos^{-1} x$.

Q03. Let R be the relation defined on Q (set of rational numbers) as $a\,R\,b \Leftrightarrow |a-b| \le \frac{1}{2}$.
Then state the reason why R is not a transitive relation?

OR

Discuss the surjectivity of $f : Z \to Z$ given as $f(x) = 3x + 2 \ \forall \ x \in Z$.

Q04. Find the value of $|A||adj.A|$ if, $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

Q05. If order of A, B and C are 4×3, 5×4 and 3×7 respectively then, find the order of $C'(A' \times B')$.

OR

Write the 'sum of cofactors' of element '13' and '1' in $\begin{vmatrix} 10 & 19 & 2 \\ 0 & 13 & 1 \\ 9 & 24 & 2 \end{vmatrix}$.

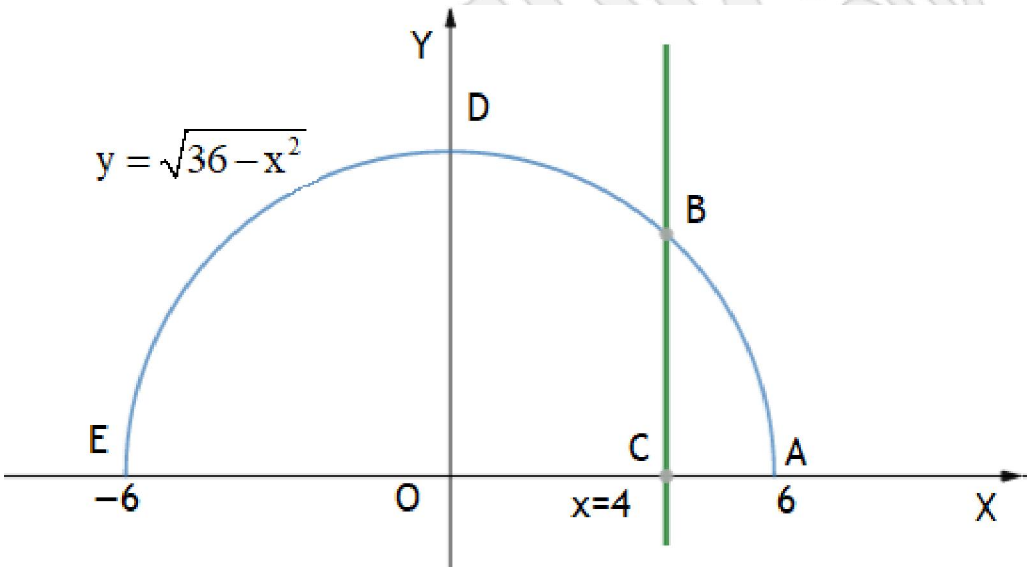
Q06. It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval [0, 2]. Find the value of 'a'.

Q07. Evaluate : $\int \frac{\sqrt{\tan x^2}}{\sin x^2 \cos x^2} x \, dx$.

OR

Evaluate : $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$.

Q08. Using integration, find the area of the smaller region as depicted in the diagram below :



Q09. Solve the differential equation $\frac{dy}{dx} + 2xy = y$ and express the result in the form of $y = f(x)$.

Q10. What is $a \in R$, such that $|a\vec{x}| = 1$, where $\vec{x} = \hat{i} - 2\hat{j} + 2\hat{k}$?

Q11. If $|\vec{a}| = 2, |\vec{b}| = 2\sqrt{3}$ and $\vec{a} \perp \vec{b}$, then write the value of $|\vec{a} + \vec{b}|$.

OR

Let $\vec{a} = \hat{i} + x\hat{j}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$ be two vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 2. Then, find the value of x.

Q12. X and Y are two points with position vectors $3\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively. Write the position vector of a point Z which divides the line segment XY in the ratio 2 : 1 externally.

Q13. If the product of the distances of the point (1, 1, 1) from the origin and the plane $x - y + z + k = 0$ be 5, then determine the value (s) of 'k'.

Q14. Find the vector equation of a line passing through the point (2, –3, –5) and perpendicular to the plane $\vec{r}.(6\hat{i} - 3\hat{j} + 5\hat{k}) + 2 = 0$.

Q15. If $P(\text{not } A) = 0.7$, $P(B) = 0.7$ and $P(B\,|\,A) = 0.5$, then find $P(\bar{A}\,|\,\bar{B})$.

Q16. From the set {1, 2, 3, 4, 5}, two numbers 'a' and 'b' (such that, $a \neq b$) are chosen at random. Find the probability that $\frac{a}{b}$ is an integer.

Section II

Questions in this section carry 1 mark each.

Both the Case study based questions are compulsory. **Attempt any 4 sub-parts from each question 17 (i-v) and 18 (i-v)**.

Q17. Meghna has two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and 'n' black balls.

One of the two boxes, box I and box II is selected by her friend Radha at random, and then Radha draws a ball at random. The ball drawn is found to be red.

Based on the above information answer the following :

(i) Meghna notices that the probability of the red ball taken out from the box II is $\frac{3}{5}$. Then Radha asks her about the value of n. The value of 'n' is

(a) 1
(b) 3
(c) 5
(d) 6

(ii) The probability that box I is selected given that the ball drawn is found to be red, is

(a) $\frac{3}{5}$

(b) $\frac{2}{5}$

(c) $\frac{1}{5}$

(d) 1

(iii) What is the probability that the ball drawn is found to be red?

(a) $\frac{5}{12}$

(b) $\frac{7}{12}$

(c) $\frac{5}{21}$

(d) $\frac{12}{5}$

(iv) Let A be the event of getting a red ball from then box. Also let E_1 and E_2 be the events that box I and box II is selected, respectively. The value of $\sum_{i=1}^{i=2} P(E_i \mid A)$ is

(a) 1

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) 0

(v) Refer to (iv) part. The value of $\sum_{i=1}^{i=2} P(E_i)$ is

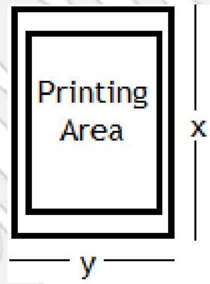
(a) 0

(b) $\frac{1}{2}$

(c) 1

(d) $\frac{1}{10}$

Q18. There is a local printing press, whose owner is given a bulk order for printing of a magazine by a school of the same locality. He shows variety of pages to school administration.
Following is the pictorial description for a particular page, selected by school administration.



The total area of the page is 150 cm^2.
The combined width of the margin at the top and bottom is 3 cm and the side 2 cm.

Using the information given above, answer the following :

(i) The relation between x and y is given by

(a) $(x-3)y=150$

(b) $xy=150$

(c) $x(y-2)=150$

(d) $(x-2)(y-3)=150$

(ii) The area of page where printing can be done, is given by

(a) xy

(b) $(x+3)(y+2)$

(c) $(x-3)(y-2)$

(d) $(x-3)(y+2)$

(iii) The area of the printable region of the page, in terms of x, is

(a) $156+2x+\frac{450}{x}$

(b) $156-2x+3\left(\frac{150}{x}\right)$

(c) $156-2x-15\left(\frac{3}{x}\right)$

(d) $156-2x-3\left(\frac{150}{x}\right)$

(iv) For what value of 'x', the printable area of the page is maximum?
(a) 15 cm
(b) 10 cm
(c) 12 cm
(d) 15 units

(v) What should be dimension of the page so that it has maximum area to be printed?
(a) Length $=1$ cm, width $=15$ cm
(b) Length $=15$ cm, width $=10$ cm
(c) Length $=15$ cm, width $=12$ cm
(d) Length $=150$ cm, width $=1$ cm

PART - B
Section III

Questions in this section carry 2 marks each.

Q19. Find the value of $\tan\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right)$.

OR

Simplify : $\cot^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}-\tan^{-1}\left(\frac{1-\sin x}{\cos x}\right)$, where $0<x<\frac{\pi}{2}$.

Q20. If $A=\begin{pmatrix}2x & 0\\ x & x\end{pmatrix}$ and $A^{-1}=\begin{pmatrix}1 & 0\\ -1 & 2\end{pmatrix}$, then find the value of x, $x\neq 0$.

Q21. Find the value of $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0^+} f(x)$, where $f(x)=\begin{cases}\frac{e^{1/x}-1}{e^{1/x}+1}, & \text{if } x\neq 0\\ -1, & \text{if } x=0\end{cases}$.

Hence, discuss the continuity of $f(x)$ at $x=0$.

Q22. Find the interval in which the function $f(x)=x\,e^{x(1-x)}$ is increasing.

Q23. Find : $\int_1^5 \left(|x-1|+|x-2|+|x-4|\right)dx$.

OR

Evaluate : $\int_0^{\pi}\frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx$.

Q24. Find the area bounded by $x^2=4y$, $x=4y-2$ and $y=0$.

Q25. Solve the differential equation $\frac{dy}{dx}+1=e^{x+y}$.

Q26. If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, find $(\vec{r}\times\hat{i}).(\vec{r}\times\hat{j})+xy$.

Q27. Find the coordinates of the point where the line through the points A(3, 4, 1) and B(5, 1, 6) crosses the XY-plane.

OR

Find the equation of plane containing the following lines :
$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ and $\frac{x-38}{3}=\frac{y+29}{8}=\frac{z-5}{-5}$.

Q28. The probability distribution of a random variable X, where k is a constant, is given below :

$$P(X=x)=\begin{cases}0.1, \text{ if } x=0\\ kx^2, \text{ if } x=1\\ kx, \text{ if } x=2 \text{ or } 3\\ 0, \text{ otherwise}\end{cases}.$$

Determine

(a) the value of k

(b) $P(X\le 2)$.

Section IV

Questions in this section carry 3 marks each.

Q29. Show that the function $f:R\to\{x\in R:-1<x<1\}$ defined by $f(x)=\frac{x}{1+|x|}$, $x\in R$ is one-one and onto function.

Q30. Discuss the differentiability of $f(x)=\begin{cases}x^2\sin\frac{1}{x}, \text{ if } x\neq 0\\ 0, \text{ if } x=0\end{cases}$ at $x=0$.

Q31. If $x=e^{\cos 2t}$ and $y=e^{\sin 2t}$, prove that $\frac{dy}{dx}=-\frac{y\log x}{x\log y}$.

OR

If $x^m y^n=(x+y)^{m+n}$, then prove that

(i) $\frac{dy}{dx}=\frac{y}{x}$ and

(ii) $\frac{d^2y}{dx^2}=0$.

Q32. Show that the equation of normal at any point t on the curve $x=3\cos t-\cos^3 t$ and $y=3\sin t-\sin^3 t$ is $4(y\cos^3 t-x\sin^3 t)=3\sin 4t$.

Q33. Find : $\int\left(\sqrt{\tan x}-\sqrt{\cot x}\right)dx$.

Q34. Using integration, find the area above x-axis, which is bounded by $\frac{x^2}{16}+\frac{y^2}{12}=1$, $y=0$ and the ordinates represented by both the latus-rectums of the given ellipse.

Q35. Solve : $y+\frac{d}{dx}(xy)=x(\sin x+\log x)$.

OR

If y(x) is a solution of $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx}=-\cos x$ and $y(0)=1$, then find the value of $y\left(\frac{\pi}{2}\right)$.

Section V

Questions in this section carry 5 marks each.

Q36. Find x, y, z if $A=\begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$ satisfies $A' = A^{-1}$.

OR

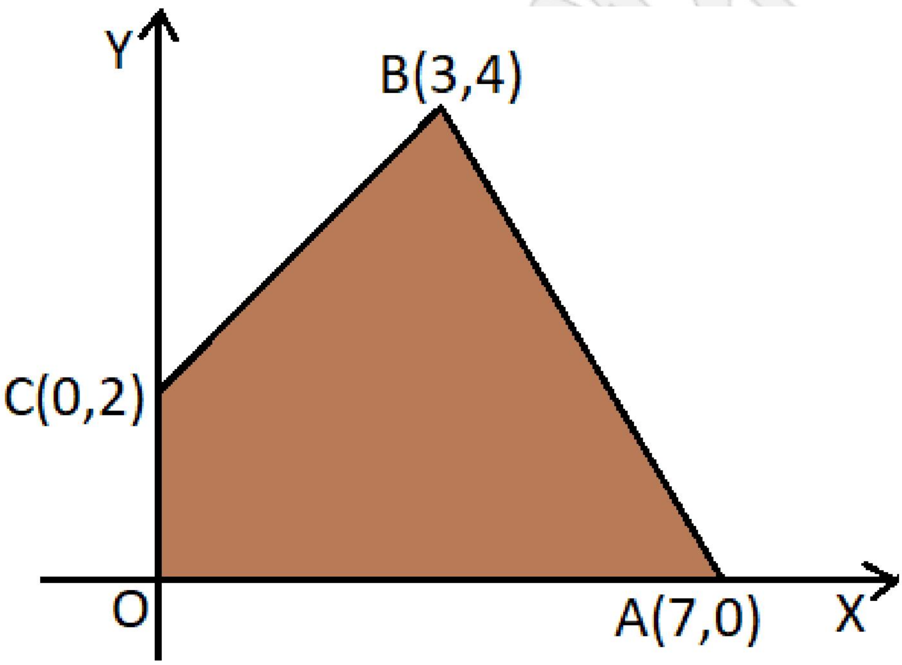
Using matrices, solve the following system of equations :

$3x+2y+z=10,$

$4x+y+3z=15,$

$x+y+z=6.$

Q37. The corner points of the feasible region determined by the system of linear constraints are as shown below :



Answer each of the following :

(i) Let $Z=5x+7y$ be the objective function. Find the maximum value of Z and, also the corresponding point at which the maximum value occurs.

(ii) Let $Z=px+y$ and $Z_A = Z_C$ then, determine the value of p.
Also, what will be the change in the value of p, if $Z=px+y$ and $Z_A = 2Z_C$?

OR

Use graphical method to solve the following linear programming :
To minimize : $Z=2x+y$
Subject to the constraints :

$x \geq 0,$

$y \geq 0,$

$4x+y \geq 80,$

$x + 5y \geq 115$,

$3x + 2y \leq 150$.

Also write the point at which maximum value of Z occurs.

Q38. A variable plane which remains at a constant distance 3p from the origin, cuts the coordinate axes at A, B and C respectively. Find the locus of the centroid of triangle ABC.

OR

Find the distance of the point $3\hat{i} - 2\hat{j} + \hat{k}$ from the plane $3x + y - z + 2 = 0$ measured parallel to the $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-1}{1}$. Also find the foot of perpendicular from the given point upon the given plane.